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Nonlinear control for linear motors with friction - Application to an inverted pendulum system

Samer Riachy, Thierry Floquet, and Jean-Pierre Richard

Abstract— **A linear motor is used here as an actuator for a cart-pendulum system. The global, upward stabilization of the inverted position is aimed at. In such an under-actuated situation, (i) constraints on the motor motion (limited length) have to be taken into account and (ii) friction effects may have a strong influence (limit cycles). A two-step path planning-plus-tracking strategy allows for dealing with constraint (i). Regarding point (ii), and since friction effects are hardly modeled, a second-order sliding mode algorithm is chosen. The resulting controller is designed without any knowledge of neither the electromagnetic circuits of the linear motor (the model is a simple gain), nor the friction models (only upper bounds for the friction forces acting on both the linear motor and pendulum are required). Experimental results show good performances in tracking and regulation, both for the swing-up and stabilization phases of the inverted pendulum.**

I. INTRODUCTION

Control of electric drives constitutes a large domain of research. When coupled to a mechanical system, as in the case of a cart-pendulum system, the modeling may result in a high order model. The resulting system becomes difficult to investigate in control. In this paper, the linear drive will be modeled in a very simplified way (*i.e.*, a linear gain) and this lack of modeling information will be compensated by a robust control design.

The cart-pendulum system has played a long-standing test-bed role in control laboratories. An interesting problem (and probably the most impressive for demonstration) is the swing-up and upward stabilization of the inverted pendulum around its unstable, top equilibrium position (say: angle $\theta = 0$). It constitutes an *under-actuated* situation, in which two outputs (translatory motion of the cart, say: x variable, and rotation angle θ of the rod) have to be controlled by means of a single actuator. Besides, such an electro-mechanical process is *nonlinear*, since the global motion presents several equilibrium points, and a simple linear model cannot represent it. Lastly, it refers to several concrete situations[1].

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[1]The damping of a mass hanged to a travelling crane refers to the bottom position (local) control [7]; The top position (local) control is encountered when balancing new two-wheel vehicles as B2 [8] or Segway [9]; The bottom-to-top –then, global– stabilization arises in particular wheel-chairs [2], [10] that can raise from a 4-wheels to a 2-wheels situation, so as to make the sited person reach a higher position.

Considering the cart-pendulum system, two main problems are encountered:

1) *Constraints on the motor motion*, which must move within a limited length, have to be taken into account. Here, it is chosen to combine a linear motor (whose high acceleration performance allows for generating high speeds within short lengths) with a two-step path planning + tracking strategy. Such a control ensures the tracking of a planned (then, admissible) reference trajectory in x, which is computed so as to be admissible with regard to the constraints, and to make the angle θ behave according to a tunable, Van der Pol-like equation. An additional closed-loop control guarantees the tracking of this reference whatever the perturbations are.
2) *Friction effects* may have a strong influence. This will be shown in the paper by considering control laws from the literature [3], the design of which was relying on a simplified model without friction. The resulting concrete application makes limit cycles appear. Here, we take it explicitly into account and, since friction effects are hardly modeled, a second-order sliding mode algorithm is chosen (for a detailed discussion on Higher Order Sliding Modes see [5] and references therein). The resulting controller is designed without any knowledge of neither the electromagnetic circuits of the linear motor, nor the friction models (only upper bounds for the friction forces acting on both the linear motor and pendulum are required).

The developed second order sliding mode controller is shown to be robust to unmodeled nonlinear phenomena and disturbances. In addition good performances are obtained in stabilization, trajectory tracking, swing-up and balancing of an inverted pendulum coupled to the linear motor.

II. TRACKING OF A MODIFIED VAN DER POL REFERENCE SIGNAL

In the next section a second order sliding mode controller is developed for the linear motor to swing-up and balance an inverted pendulum. In order to swing the inverted pendulum, the linear motor needs to follow a modified Van der Pol reference signal [4].

The pendulum coupled to the linear motor is shown in Fig.1 and is governed by (1)-(3)

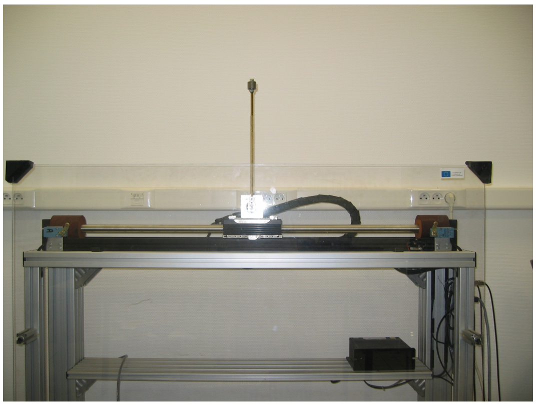

Fig. 1. Inverted pendulum located at the LAGIS laboratory.

$$(M+m)\ddot{x}+ml\sin\theta\ \dot{\theta}^2-ml\cos\theta\ \ddot{\theta}=\tau+w_1(t)-\psi(\dot{x}), \quad (1)$$

$$\frac{4}{3}ml^2\ddot{\theta}-ml\cos\theta\ \ddot{x}-mgl\sin\theta=w_2(t)-\varphi(\dot{\theta}) \quad (2)$$

$$\tau=KV \quad (3)$$

where x is the cart position, θ is the angular deviation of the pendulum from the vertical, M is the cart mass, m is the rod mass, l is the distance to the center of mass of the pendulum, g is the gravitational acceleration, τ is the force generated by the linear motor, $w_1(t), w_2(t)$ are external disturbances, $\psi(\dot{x})$ and $\varphi(\dot{\theta})$ are friction forces, affecting the linear motor and the pendulum, respectively. Note that the linear motor is modeled in (3) as a linear gain K acting on the input voltage V.

The general idea of the swing-up control is inspired from [4], while the local control relies on [6]. For lack of space, only general ideas are given here. The modified Van der Pol oscillator is given by:

$$\ddot{z}+\varepsilon\left[(z^2+\frac{\dot{z}^2}{\mu^2})-\rho^2\right]\dot{z}+\mu^2 z=0, \quad (4)$$

This modification possess a stable limit cycle (a sinusoidal one), the advantage of which is to be expressible in the explicit form

$$z^2+\frac{\dot{z}^2}{\mu^2}=\rho^2 \quad (5)$$

Our objective is to design a controller that causes the actuated part of the cart-pendulum system, i.e. the linear motor, to track a trajectory generated by the modified Van der Pol equation (4), i.e:

$$\lim_{t\to\infty}[z(t)+x(t)]=0, \quad (6)$$

while attenuating the effect of the friction forces, external disturbances, and unmodeled dynamics.

Consider the sliding variable:

$$y(t)=z(t)+x(t), \quad (7)$$

that combines the actuated position $x(t)$ of the system and the reference variable $z(t)$ governed by the modified Van der Pol equation (4). The control problem is to drive the system output (7) onto the surface $y=0$ in finite time and maintain it there in spite of the friction forces, external disturbances, and unmodeled dynamics affecting the system.

By differentiating (7) twice, one obtains:

$$\begin{aligned}\ddot{y} &= u+\frac{3\cos\theta}{4lJ}\left[w_2(t)-\varphi(\dot{\theta})\right]+\frac{1}{J}\left[w_1(t)-\psi(\dot{x})\right]\\ &\quad -\varepsilon\left[(z^2+\frac{\dot{z}^2}{\mu^2})-\rho^2\right]\dot{z}-\mu^2 z. \quad (8)\end{aligned}$$

Relating the quasihomogeneous synthesis from [6], the following second order sliding mode control law

$$\begin{aligned}u &= \frac{3\varphi_v\cos\theta}{4lJ}\dot{\theta}+\frac{\psi_v}{J}\dot{x}+\varepsilon\left[(z^2+\frac{\dot{z}^2}{\mu^2})-\rho^2\right]\dot{z}+\mu^2 z\\ &\quad -\alpha sign(y)-\beta sign(\dot{y})-hy-p\dot{y} \quad (9)\end{aligned}$$

with the parameters such that

$$h,p\geq 0,\ \alpha-\beta>\frac{3(\varphi_c+N_2)}{4lJ}+\frac{\psi_c+N_1}{J} \quad (10)$$

is proposed.

It was demonstrated in [4] that the resulting quasihomogeneous system (8), (9) with the parameter subordination (10) is finite time stable regardless of unmodeled dynamics, friction forces and uniformly bounded external disturbances affecting the system. So, after a finite time, the cart-pendulum system evolves on the second order sliding manifold i.e. on the zero dynamics surface $y=0$. The control objective is thus achieved in finite time (non asymptotic).

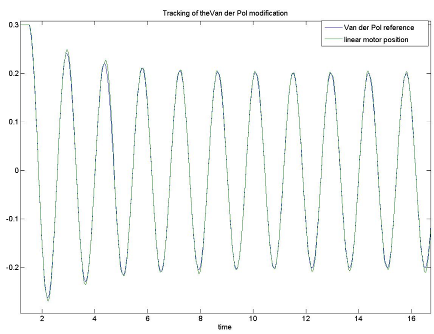

Fig. 2. Experimental tracking of the Van der Pol reference signal (reference and ouput).

Fig. 2 shows experimental results of the linear motor (green) tracking the Van der Pol reference (blue). It can be seen that the linear motor position starts from the Van der Pol reference signal, this initial condition is chosen for practical reason (short cart length).

III. Swing up control and stabilization

To swing the pendulum up from the bottom position to the top position, the orbitally stabilizing synthesis (4), (8), (9) is applied to pump into the system as much energy as required to approach the basin of attraction of a locally stabilizing controller. Switching to a local second order sliding mode controller (a detailed discussion can be found in [6]) completes the objective of swing-up and balancing of the inverted pendulum.

A. Cart-pendulum prototype

In order to study the performance of the proposed synthesis experimental tests were made. The real parameters of the laboratory cart-pendulum system are given in table I.

TABLE I

PARAMETERS OF THE CART-PENDULUM.

Notation	Value	Units
M	3.4	kg
m	0.147	kg
l	0.175	m
ψ_v	8.5	$N\cdot s/m$
φ_v	0.0015	$N\cdot m\cdot s/rad$
ψ_c	6.5	N
φ_c	0.00115	$N\cdot m$

B. Swinging controller design

To swing the pendulum up from the downward position to the upright position, the orbitally stabilizing synthesis (4), (8), (9) is applied to pump into the system as much energy as required to approach a homoclinic orbit with the same energy level as the one corresponding to the desired equilibrium point. The idea is inspired from [3] which consists of driving a frictionless cart-pendulum to its homoclinic orbit obtained by zeroing both the total energy of the system and the cart velocity. The model used in this paper is that of a real life cart-pendulum, having friction both on the cart and the pendulum axis. So, we speak of a quasi-homoclinic orbit which still gives large oscillations of the pendulum from the downward position to the upright one which is the main interest for swing-up. In experiments, the total energy of the cart-pendulum

$$\begin{aligned} E(q,\dot q) &= \frac{1}{2}(M+m)\dot x^2 - ml\dot x\dot\theta\cos\theta + \frac{2}{3}ml^2\dot\theta^2 \\ &\quad +mgl(\cos\theta-1) \end{aligned} \tag{11}$$

is calculated at each instant when the cart is at its maximal displacement (i.e. zero cart velocity) on the modified Van der Pol trajectory. Starting from a negative value $E_1 = -2mgl$ when the system is at rest on the downward pendulum position, the energy increases due to (9) until it comes slightly greater than zero (to take into account friction forces)

$$E_0 \succeq 0. \tag{12}$$

The swinging controller is then switched off, and the pendulum is left to travel on its quasi-homoclinic orbit toward the upright position. Being crucial to a successful swing up, this is achieved by tuning both the controller parameters $\alpha,\ \beta,\ h,\ p$ and the reference parameters $\varepsilon,\ \rho,\ \mu$ of the Van der Pol modification (4). Appropriate values of the parameters to be tuned are carried out in successive numerical experiments. In our experimental study, the controller gains in (9) were set to $\alpha = 20rad/sec^2, \beta = 5rad/sec^2,\ h = 0,\ p = 0$ whereas the reference parameters were tuned to $\varepsilon = 10\ [rad]^{-2}s^{-1},\ \rho = 0.2\ rad,\ \mu = 0.7\times 2\pi\ s^{-1}$.

C. Locally stabilizing controller design

The locally stabilizing controller developed in [6] is defined as:

$$\begin{aligned} \tau &= \frac{D\cos\theta}{\left[3+8l\lambda_2\dot\theta\sin\theta\right]}\times \\ &\quad \left[-\mu(\theta,\dot\theta)-\alpha_1 sign(s)-\beta_1 sign(\dot s)-h_1 s-p_1\dot s\right], \\ &\quad \alpha_1,\beta_1,h_1,p_1\geq 0 \end{aligned} \tag{13}$$

with

$$\begin{aligned} \mu(\theta,\dot\theta) &= 2\frac{\tan\theta}{\cos^2\theta}\dot\theta^2 + \left[\frac{3+8l\lambda_2\dot\theta\sin\theta}{\cos\theta}\right]\left[\frac{(M+m)g-ml\cos\theta\ \dot\theta^2}{D}\right] \\ &\times\tan\theta + \lambda_1\left(g+\frac{4}{3}l\frac{\dot\theta^2}{\cos\theta}\right)\tan\theta + \lambda_2\left(g+\frac{4}{3}l\frac{\dot\theta^2(1+\sin^2\theta)}{\cos\theta}\right)\frac{\dot\theta}{\cos^2\theta}, \end{aligned} \tag{14}$$

$$D = l(4M+m+3m\sin^2\theta),\ s=\tan\theta-\lambda_1\omega-\lambda_2\dot\omega,\ \lambda_1,\lambda_2>0 \tag{15}$$

$$\omega = x-\frac{4l}{3}\ln\left(\frac{1+\sin\theta}{\cos\theta}\right),\quad |\theta|<\frac{\pi}{2}, \tag{16}$$

is tested in an experimental study. Quite impressive experimental results are obtained for the local stabilization of the pendulum about the upright position with the controller parameters $\alpha_1 = 30\ m/s^2,\ \beta_1 = 7.5\ m/s^2,\ h_1 = 0,\ p_1 = 0,\ \lambda_1 = 2\ 1/m,\ \lambda_2 = 1\ s/m$. The results can be observed in Subsection III-E where the proposed controller is introduced into a hybrid synthesis of swinging the Cart-Pendulum up and balancing it about the vertical.

D. Hybrid Controller Design

In order to accompany swinging the pendulum up by the subsequent stabilization around the upright position, the swinging controller, presented in Subsection III-B, is turned off once the system reaches the corresponding homoclinic orbit, and then the locally stabilizing controller from III-C is turned on whenever the pendulum enters the basin of attraction, numerically found for the latter controller. The resulting hybrid controller, thus constructed, moves the inverted pendulum, located on the cart, from its downward position to the upright position and stabilizes it about the vertical whereas the cart is stabilized at the desired endpoint. While being not studied in details here, the capability of the closed-loop system to reach the homoclinic orbit and entering the attraction basin of the locally stabilizing controller is supported by experimental results.

E. Experimental results

Experimental results are plotted in Figs. 3-4. They show that the pendulum starts from the "stable" position π, oscillates until it reaches the unstable equilibrium position 0 and remains there.

Other approaches have been implemented and compared (see Fig. 5 corresponding to [3]). It can be seen that the swing-up controller was not able to drive the pendulum close to its top position. This is mainly due to the neglected

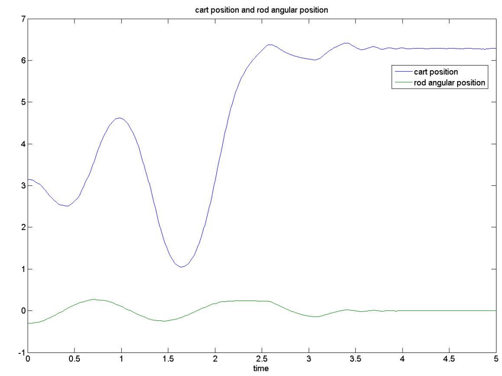

Fig. 3. Cart position and rod angular position versus time.

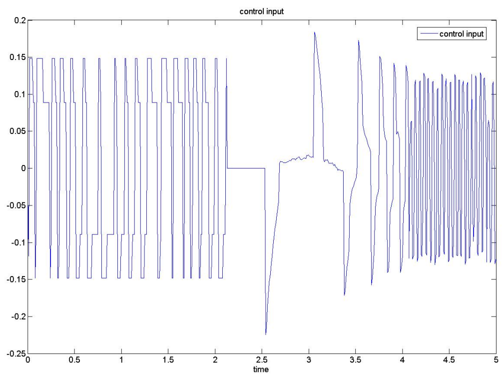

Fig. 4. Control input versus time.

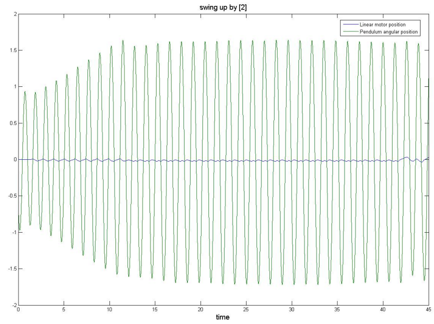

Fig. 5. Experimental results for the control designed in [3] (cart position and pendulum angular position).

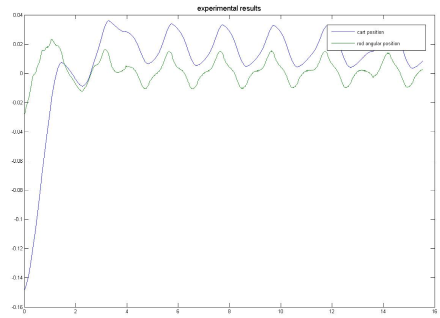

Fig. 6. Experimental results for linear static state feedback local stabilization [1] (cart position and pendulum angular position).

friction effects. A linear static state feedback has also been implemented for local upright stabilization (see Fig. 6). Large amplitude oscillations (limit cycles) occur.

A drawback of sliding mode control could be the high frequency oscillations in the control signal (see Fig. 4) called chattering. There is a huge literature on how to attenuate this problem. A simple solution, which we used here, is to replace the *sign* function in (9) by a continuous one ($arctan$).

IV. Conclusions

Orbital stabilization of a cart-pendulum system, presenting a simple underactuated (two degrees-of-freedom, one actuator) manipulator with friction, was under study. The system was constrained to track a modified Van der Pol oscillator that possesses a stable limit cycle, governed by a standard tunable linear oscillator equation. A quasihomogeneous second order sliding mode based control synthesis was utilized to design a variable structure controller that drives the pendulum to a desired zero dynamics manifold in finite time and maintains it there in sliding mode in spite of the presence of external disturbances. Once the cart-pendulum reached a sufficient level of energy, the control was switched to a locally stabilizing quasihomogeneous controller, solving the problem of moving the pendulum from its stable downward position to the unstable inverted position and stabilizing it about the vertical.

Capabilities of the quasihomogeneous synthesis and its robustness against friction forces and external disturbances were successfully illustrated on an underactuated cart-pendulum testbed and it is hoped to suggest a practical framework for orbital stabilization of underactuated manipulators.

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